Wave modelling in the Adriatic

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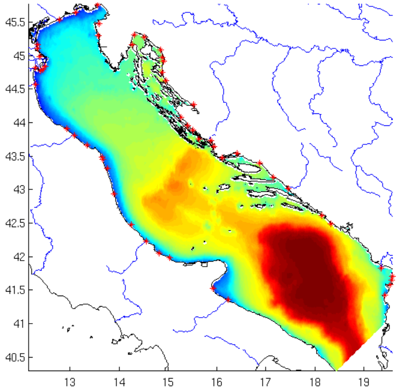
I. Hindcasting the state of the Adriatic Sea

Bathymetry and rivers of the Adriatic Sea

- The bathymetry varies a lot from 1200m to 50m.
- The island structure on the Croatian side is quite complex.
- River inflow is more important than in other parts of the Mediterranean.



- Significant inflow/outflow occurs at the Ottranto strait and generates the highest tides of the Mediterranean.
- Two winds Bora and Sirocco dominate the general circulation.

Chosen forcing information

- The chosen modelization of the Adriatic Sea uses the atmospheric forcing fields from `DHMZ` using the `ALADIN` model (sea surface pressure, temperature, humidity, rain, cloud factor, short wave radiation).
- For river forcing, we used:
  - Hourly measurements for Po river and Neretva river.
  - Daily flux measurements for 9 other rivers and temperature for 5 more.
  - For other Italian rivers, we used climatological information from Raicich, 1994. For other Croatian rivers we rescale according to Neretva inflow.
  - For temperature we took nearest river.
- We used an initial state obtained from `AREG` which is an operational model using a modification of `POM`.
- At the open boundary of the Ottranto strait, we used forcing from the `AREG` model.

The ROMS model

The ROMS model (by Hernan Arango) is a finite difference model that solves the Eulerian primitive equation in curvilinear coordinates.

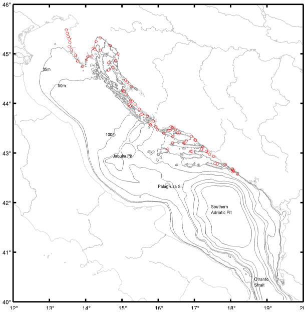
$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z} = fv-\frac{1}{\rho}\frac{\partial p}{\partial x}+F_{diss,x}+F_{wave,x}+F_{surf,x}+F_{bottom,x}$$

The ROMS model:

- uses the hydrostatic and Boussinesq approximations,
- uses sigma-coordinates for the vertical discretization,
- uses the split-explicit method in order to resolve fast surface waves with a barotropic model,
- has a variety of high order schemes for momentum advection, tracer advection, horizontal pressure gradient, etc.,
- has infrastructure for coupling with other models (SWAN, WRF, etc.),
- has 4DVAR assimilation capabilities (not used here),
- has biological and sediment sub models (not used here).

Available measurements

- Satellite AVHRR measurements sea surface temperature are available every few hours but are affected by clouds.
- Daily Medspiration synthetic data sets at 2km resolution of foundation temperature are created from various measurements and model output. `RMSE` is about 0.4*deg*.
- *In situ* CTD measurements available from cruises (Nov 2007, Mar, Jun, Jul 2008) with a priori insignificant error.



Results (CTD)

- One problem is that CTD measurements are done near the coast, exactly where the model is expected to be bad.
- For the CTD we found following results:

Mean cruise date	Temperature (deg)		Salinity (PSU)	
	RMSE	ME	RMSE	ME
01-11-2007	1.18	0.84	0.47	-0.17
20-03-2008	0.61	0.33	0.69	-0.30
01-06-2008	0.92	-0.21	0.92	-0.44
01-07-2008	1.38	-0.37	0.79	-0.41

- Large error for November 2007 is explained by the time of spin up of the model in august 2008.
- Summer is generally more difficult to model appropriately due to stronger stratification.
- Also problematic for modelling are narrow straits between islands.

Results (Medspiration)

- No Medspiration products were available for April and May.
- Foundation temperature is compared with model temperature at 3 UTC.
- For Medspiration we found following results for `RMSE`:

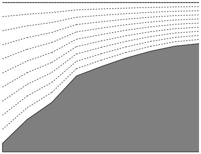
Month	North	Middle	South	Whole
January	0.86	0.82	0.73	0.79
February	0.87	0.63	0.63	0.69
March	0.70	0.57	0.52	0.58
June	1.33	0.82	0.80	0.96
July	0.89	0.93	0.95	0.93
August	0.68	0.81	0.98	0.85
September	1.09	1.00	1.02	1.03
October	0.57	0.79	0.70	0.71
Whole	0.90	0.79	0.80	0.82

- Mean error is generally very small except in June where it reaches $-0.40deg$.
- The Medspiration product itself is not a measurement and when no `AVHRR` scene is available, it then uses microwave sensors.

II. Bathymetry smoothing and nesting

σ-coordinate and the bathymetry

- The bathymetry of the ocean is varying from point to point and one standard way to deal with it is to use σ-coordinates (Phillips, 1957) as in `ROMS`.



- On every cell e of bathymetry $h(e)$, choose a number N of vertical levels $h(e, k)$ for $1 \leq k \leq N$ with $h(e, 0) = -h(e)$ and $h(e, N) = 0$.
- The differentiation rule of functions in σ-coordinate is
$$\left.\frac{\partial f}{\partial x}\right|_z = \left.\frac{\partial f}{\partial x}\right|_\sigma + \frac{\partial h}{\partial x}\frac{\partial f}{\partial \sigma}$$
- This creates a problem for horizontal derivatives, which become a difference of two terms.
- Hence one has to modify the bathymetry in order to use it in oceanic models having fixed number of vertical levels.

Bathymetry smoothing problem

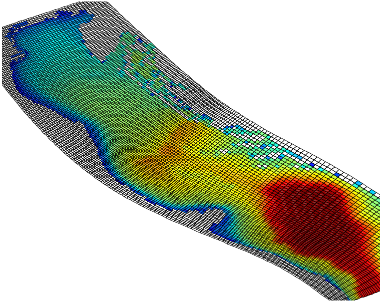
- One way to measure the effect of large bathymetry differences is the roughness factor rx_0:

$$rx_0(h, e, e') = \frac{|h(e) - h(e')|}{h(e) + h(e')} \text{ for two adjacent cells } e, e'$$

- It turns out that this factor can be optimized by using the theory of linear programming and thus one can get a bathymetry with $rx_0(h, e, e')$ small and $|h - h^{real}|$ as small as possible.
- M. Dutour Sikirić, I. Janeković, M. Kuzmić, *A new approach to bathymetry smoothing in sigma-coordinate ocean models*, Ocean Modelling **29** (2009) 128–136.
- Since then divide and conquer approach were devised that increased the speed and optimization strategies for other measure of the error.
- Also introduced were nested grid procedures.

Nesting of grids

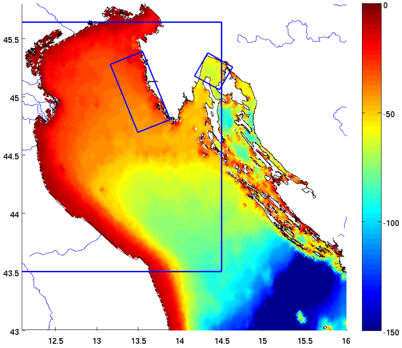
- Finite difference grids allow to use high order conservative schemes over the domain with high speed and precision.
- But the major problem of such methods is that the grid size is more or less uniform over the whole domains:



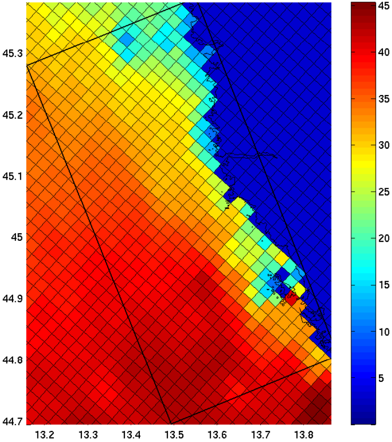
Moreover, if the grid size varies then this introduce some numerical errors. So, a common practice is to use uniform grids.

Interpolation and one way nesting

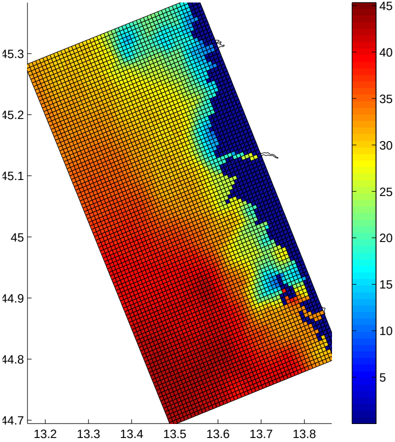
- The idea is to have one model running at a coarse resolution over the whole domain.
- Then we have a model running on a finer grid on a subregion of interest.
- The information of the coarse grid is used to force the finer grid.
- The procedure can be done coherently with the `SWAN` model.



Interpolating bathymetry between grids

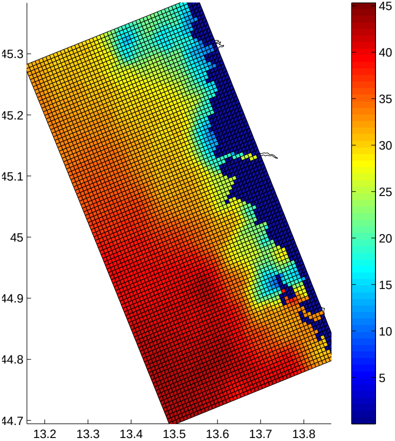


Bathymetry of 2*km* grid

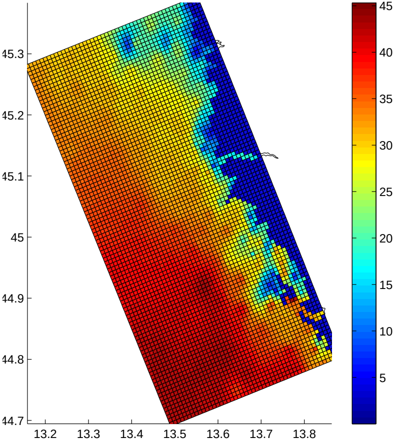


Bathymetry of 2*km* grid interpolated to 700*m*

Interpolated bathymetry versus averaged

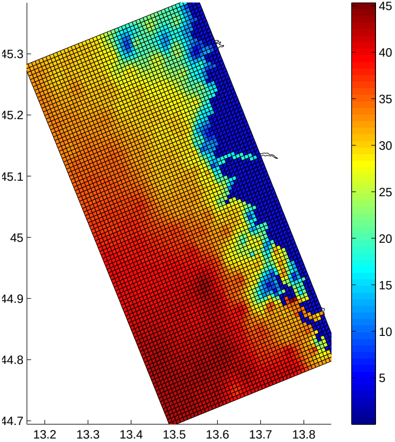


Bathymetry of $2km$ grid interpolated to $700m$

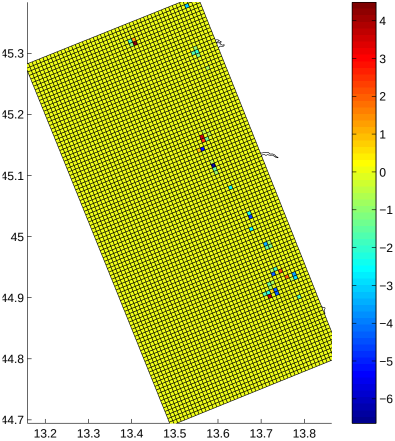


Raw bathymetry averaged to $700m$

LP input and the effect of LP smoothing



Bathymetry of *LP* input



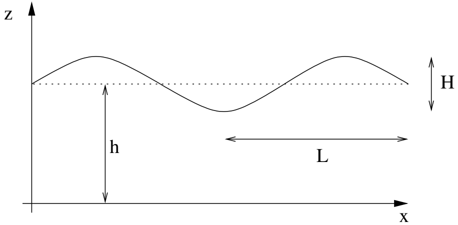
Differences introduced by *LP*

III. Wave modelling

Linear water wave theory I: setting

The basic assumptions of linear water wave theory are:

- The velocities and free surface elevation are small.
- The depth h is uniform.
- The fluid has no viscosity, is incompressible and irrotational.



We also call T the wave period and define $\omega = \frac{2\pi}{T}$ and $k = \frac{2\pi}{L}$.

Linear water wave theory II: solution

Under the above assumptions, it is possible to derive explicit formulas for the free surface η and velocity (u, w)

$$\eta = \frac{H}{2}\sin\left(2\pi\left(\frac{x}{L} - \frac{t}{T}\right) + \phi\right) \text{ and } (u, w) = \left(\frac{\partial\phi}{\partial x}, \frac{\partial\phi}{\partial z}\right)$$

with

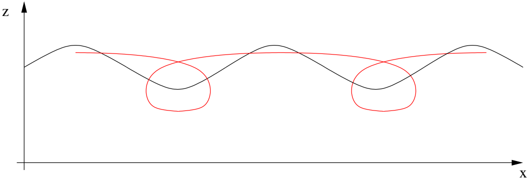
$$\phi(x, z) = \frac{gH}{2\omega}\frac{\cosh(k(z+h))}{\cosh(kh)}\sin\left(2\pi\left(\frac{x}{L} - \frac{t}{T}\right) + \phi\right).$$

The wave period T and wave length L satisfies the dispersion relation

$$\omega^2 = gk\tanh(kh)$$

Linear water wave theory III: Stokes drift

- The above expressions for velocities are given in an Eulerian fixed frame.
- Integrated over a long time, the velocities have zero mean.
- But if one considers a Lagrangian viewpoint, i.e. follows the particles then there is a movement:



- A simplified formula for the resulting velocity is the Stokes drift

$$U_S = \frac{1}{2}\left(\frac{\pi H}{L}\right)^2 \frac{\omega}{k} \frac{\cosh(2k(z+h))}{\sinh^2(kh)}$$

Stochastic wave modelling

- Oceanic models are using grids (structured or unstructured) of size a few hundred meters to simulate the ocean
- But oceanic waves have a typical wavelength of a few hundred meters which means that we cannot resolve the phase and amplitude of oceanic waves exactly.
- But if one uses phase averaged models and uses stochastic assumptions then it is possible to model waves by a spectral wave energy density $N(\mathbf{x}, \mathbf{k})$
- This density satisfies a wave action equation which represents the advection in geographical, frequency/directional space and evolution by wind input, dissipation and nonlinear interactions:

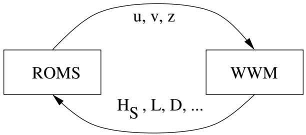
$$\frac{\partial N}{\partial t} + \nabla_x((c_g + U_A)N) + \nabla_k(\dot{k}N) + \nabla_\theta(\dot{\theta}N) = S_{tot}$$

with

$$S_{tot} = S_{ds} + S_{in} + S_{nl3} + S_{nl4} + S_{bot} + S_{break}$$

Wave coupling

- Wave models use surface currents for the advection of wave energy and the free surface enters into the dispersion relation.
- On the other hand oceanic model can use wave information to:
  - Compute the Stokes drift (current induced by waves, a nonlinear effect).
  - Compute the wave radiation pressure term in the primitive equation.
  - Improve the computation of the surface stress, turbulence.
  - Be used in sediment transport models.
- Thus it makes sense to have oceanic and wave models coupled both ways. We chose to work with the `ROMS` model (a finite difference model) and the `WWM` model (a finite element model by Aron Roland).



Longuet-Higgins, Mellor and Ardhuin theories

- (Longuet-Higgins, 1953) derived an expression for the barotropic stress induced by waves.
- (Mellor, 2003) proposed some expression for the baroclinic stress but some incoherent results were obtained with it.
- (Ardhuin, 2007) proposed a new set of equations with the basic prototype for the tracer equation:

$$\frac{\partial T}{\partial t} + (u + u_S)\frac{\partial T}{\partial x} + (v + v_S)\frac{\partial T}{\partial y} + (w + w_S)\frac{\partial T}{\partial z} = S_T$$

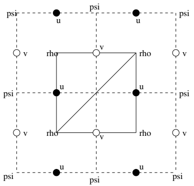
  with $(u, v, w)_S$ being the Stokes drift and S_T the source of the tracer.
- The boundary condition for momentum changes from $u = 0$ to $u = -u_S$.

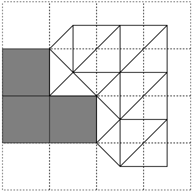
Numerics of the coupling

- The mathematical expressions occurring in wave modelling are for example
$$\frac{\cosh(2kz)}{\sinh(2kh)}$$
- This kind of function is very singular. Their large values are concentrated on the surface. On the other hand it satisfies a specific integral property:
$$\frac{1}{h}\int_0^h \frac{\cosh(2kz)}{\sinh(2kh)} dz = \frac{1}{2kh}$$
which has to be reproduce in the model.
- The solution that we choose is for every vertical cell of the model, to put the average value at the relevant point.

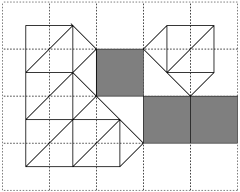
Grid subdivizion schemes

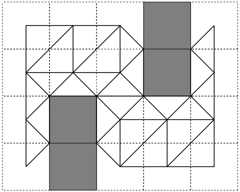
- Our standard interpolation strategy is to subdivide the squares in two triangles. Then near the coast, we add some more triangles.





- Those additional triangles allow us to respect the straits and isthmus of the original grid.

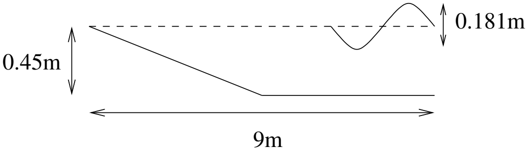




- But the system also allows some finite element grid to be used. The relevant interpolation is done in `FORTRAN`.

Shoaling idealized test case I

- For the shoaling idealized test case, a wave of period $1.5s$ arrives on a beach and breaks:



- The significant wave height satisfies to the two constraints:

$$\begin{aligned} H_S^2 c_g &= Cst \text{ if no wave energy dissipation,} \\ H_S &\leq c_B(h+z) \text{ with } c_B = 0.415. \end{aligned}$$

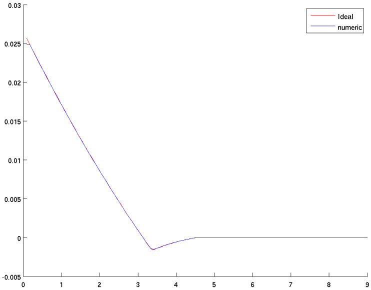
- The stress balance equation is

$$\frac{\partial S_{xx}}{\partial x} = -\frac{1}{h+z}\frac{\partial h}{\partial x}$$

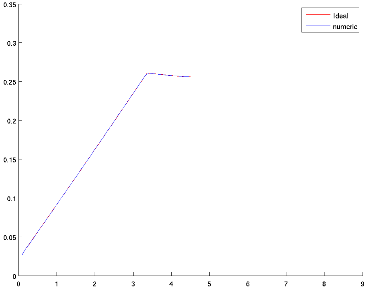
with S_{xx} the Longuet-Higgins potential, h the depth and z the free surface.

- The equation system can be solved very accurately.

Shoaling idealized test case II



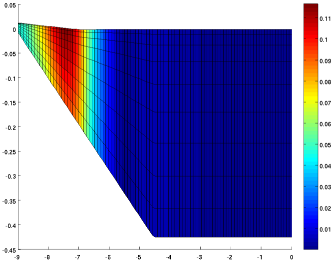
Free surface



Significant wave height

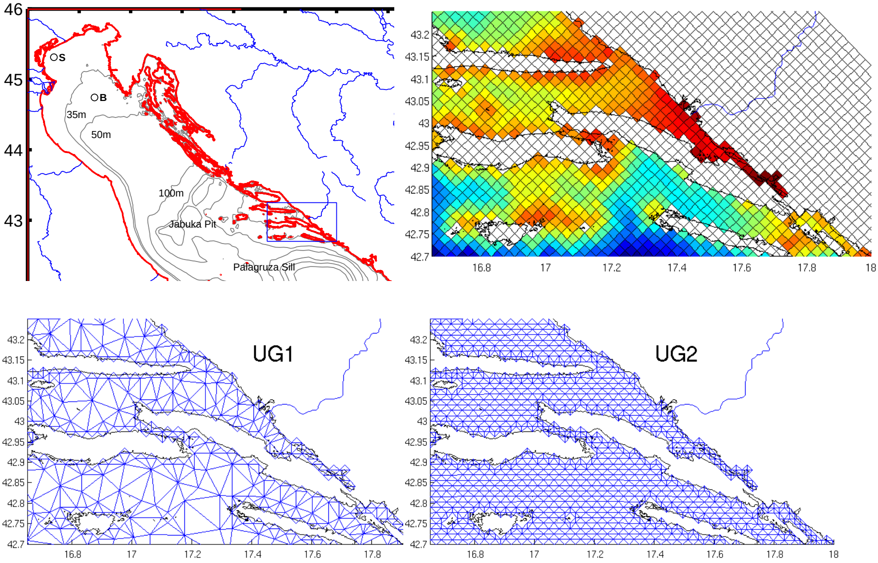
Visser's idealized test case

- Another important test case is Visser's test case where the waves are arriving obliquely on the beach. A longshore current is induced by the waves and it is balanced by dissipation in the model.
- In order to adequately model such situations, we introduce ideal grid, that is grids where the model does not see the whole coordinate system:
  - Input contains triangle area, list of nodes and node depth.
  - Differences of coordinates between nodes of each triangles.



Longshore current

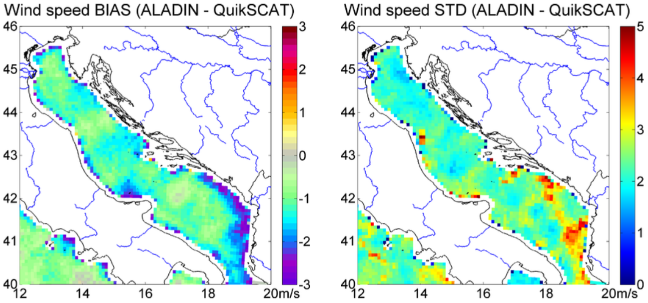
Grids of the Adriatic



Comparison with QuikSCAT (by Igor Tomažić)

- QuikSCAT scatterometer provides sea surface ($10m$) wind field at a $12.5km$ resolution.
- Instrument specification gives zero bias and $\mathtt{RMSE}$ $2m/s$ and $20deg$ for magnitude and direction. Validation studies with *in situ* data in coastal region ($< 80km$) show larger error ($0.93 \pm 1.83m/s$) and ($4.71 \pm 31.15deg$) (Tang et al., 2004).
- Validation of ALADIN data with QuikSCAT data shows good agreement and relatively small error:

  speed: $-1.15 \pm 2.50m/s$, direction: $-4.16 \pm 38.14deg$



Comparison with wave stations I

Comparison with station S

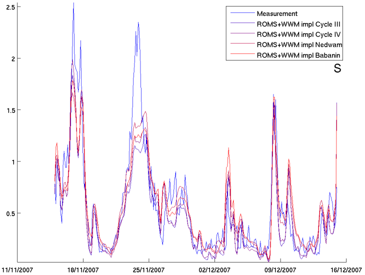
parameterization	Significant wave height			Mean wave length			Mean direction	
	RMSE	ME	Corr	RMSE	ME	Corr	ME	RMSE
Cycle III	0.26	0.09	0.87	24.55	20.55	0.68	-1.77	60.51
Cycle IV	0.27	0.10	0.86	19.43	15.83	0.72	4.29	56.45
Nedwam	0.23	0.01	0.88	20.17	14.54	0.73	7.83	55.74
Babanin	0.25	0.01	0.86	20.23	16.29	0.73	14.62	66.50

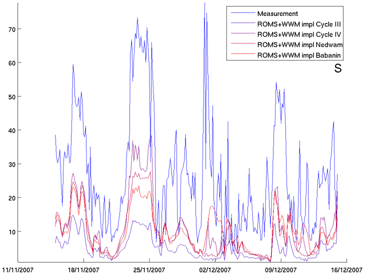
Comparison with buoy B

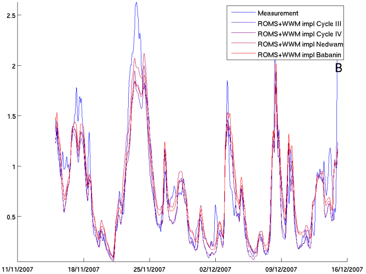
parameterization	Significant wave height			Mean wave length			Peak direction	
	RMSE	ME	Corr	RMSE	ME	Corr	ME	RMSE
Cycle III	0.24	0.10	0.90	17.17	15.30	0.70	-0.98	61.58
Cycle IV	0.23	0.09	0.90	10.60	8.73	0.79	3.94	64.09
Nedwam	0.20	-0.01	0.92	11.42	9.48	0.76	2.16	63.48
Babanin	0.23	-0.01	0.89	12.74	11.00	0.77	5.71	60.09

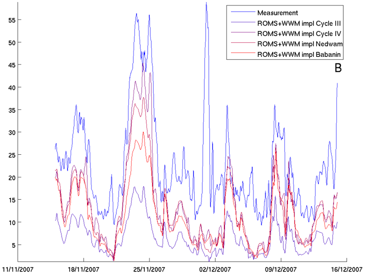
Those runs are done with the implicit scheme of `WWM` and the finite difference grid (`UG2`, 32810 nodes) and not the `FEM` grid (`UG1`, 4896 nodes)

Comparison with wave stations II





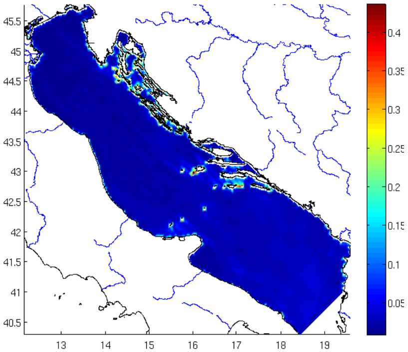




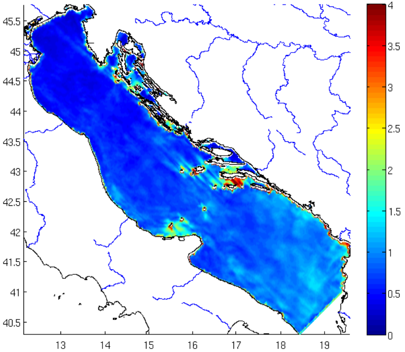
Significant wave height

Mean wave length

Mean absolute error between grid UG1 and UG2 I



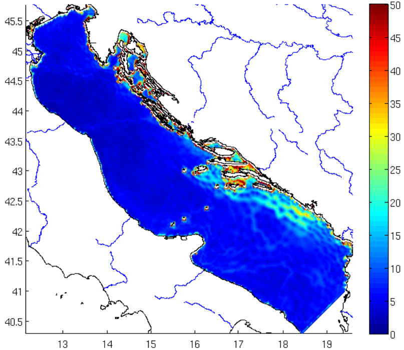
Significant wave height



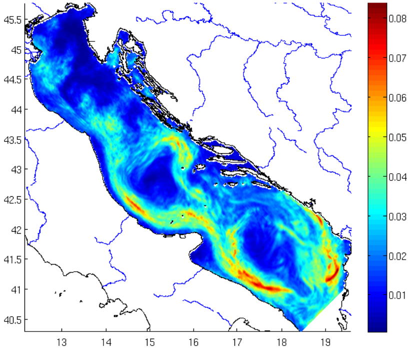
Mean wave length

- The grid impact on Significant wave height is very small.
- It is larger for Mean wave length, but still reasonable.

Mean absolute error between grid UG1 and UG2 II



Mean wave direction

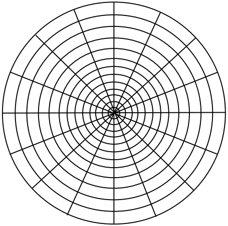


Surface current

- Wind direction is more unstable when the waves are smaller.
- For surface current, we see that the Stokes drift is fairly important for the Western Adriatic Current.
- The future appears to be to go to unstructured models (Work in preparation with Y. Zhang and A. Roland on coupling of `SELFE` and `WWM`).

Possible extensions of the coupling

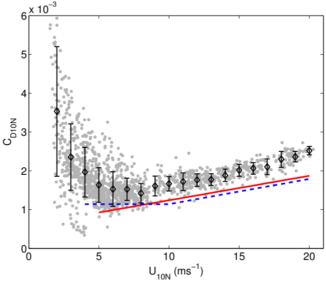
- Right now `ROMS` is using the mean information from the wave spectrum: Significant wave length, Mean wave length and Mean direction.
- But the wave model provides the wave spectrum at every point with MSC frequencies, MDC directions (total $MSC \times MDC$ values).



- Thus it would be a good idea to express the Stokes drift as integral over the spectrum.
- The `ROMS` code is also wrong for Lagrangian drifters: it does not use the Stokes drift for the advection of particles or floaters.

Surface turbulence, stress and dissipation I

- The basic fundamental mechanism is that the wind blows over the sea ...
- ... transfers momentum to the surface of the ocean ...
- ... which is diffused deeper by turbulence.
- The surface stress is expressed as $\tau = \rho_{air} C_D U_{10m}^2$ with C_D the drag coefficient. The problem is to estimate C_D.



(O'Campo Torres et al. GRL 2009) for wind opposed to Swell in the Pacific near the Mexican coast.

Surface turbulence, stress and dissipation II

- On the other hand wave models have to model the wind input to the wave and any such modelization (Cycle III, Cycle IV, Nedwam, Babanin, Ardhuin, etc) also determines the stress applied to currents.
- When waves dissipate, almost all its momentum goes to current (mostly at the surface) thus creating some memory effects.
- But the energy dissipated by waves acts as a source of turbulent kinetic energy (as opposed to Craig & Banner which considers it as boundary condition).

Source P.A.E.M. Janssen and F. Ardhuin

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THANK

YOU