# The Cone 22SMET5

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## 1 The intrisic informations concerning the graph of extreme rays of 22SMET5.ext

The graph considered is made of 132 elements.
The group acting on 22SMET5.ext is Group2hmet5
Under the group action we have in fact only 6 orbits to consider.
The diameter of the graph is 2.
The list of orbits is with adjacency and Size info:

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| E1: | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 92 | 5 |
| E2: | 0 | 2 | 2 | 2 | 2 | 1 | 2 | 2 | 1 | 1 | 32 | 10 |
| E3: | 0 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 28 | 15 |
| E4: | 1 | 1 | 2 | 2 | 1 | 2 | 2 | 2 | 1 | 1 | 25 | 12 |
| E5: | 1 | 2 | 4 | 3 | 3 | 2 | 3 | 4 | 4 | 1 | 13 | 60 |
| E6: | 0 | 4 | 4 | 4 | 4 | 2 | 4 | 4 | 2 | 5 | 13 | 30 |

The representation matrix is:

| | E1 | E2 | E3 | E4 | E5 | E6 | Adj. | Size |
|---|---|---|---|---|---|---|---|---|
| E1 | 4 | 10 | 12 | 12 | 36 | 18 | 92 | 5 |
| E2 | 5 | 0 | 3 | 6 | 12 | 6 | 32 | 10 |
| E3 | 4 | 2 | 2 | 4 | 12 | 4 | 28 | 15 |
| E4 | 5 | 5 | 5 | 0 | 5 | 5 | 25 | 12 |
| E5 | 3 | 2 | 3 | 1 | 2 | 2 | 13 | 60 |
| E6 | 3 | 2 | 2 | 2 | 4 | 0 | 13 | 30 |
| Size | 5 | 10 | 15 | 12 | 60 | 30 | | 132 |

### 1.1 The local graphs

The complemented local graph are:
We consider graph with cardinal inferior or equal to 20
The complemented local graph for orbit 5 is:

| | | | |
|---|---|---|---|
| 1 | 6 | : | 2 3 5 7 8 12 |
| 2 | 6 | : | 1 5 6 10 11 12 |
| 3 | 5 | : | 1 6 7 10 12 |
| 4 | 0 | : | |
| 5 | 2 | : | 1 2 |
| 6 | 5 | : | 2 3 8 11 12 |
| 7 | 4 | : | 1 3 10 11 |
| 8 | 5 | : | 1 6 9 10 11 |
| 9 | 3 | : | 8 10 12 |
| 10 | 5 | : | 2 3 7 8 9 |
| 11 | 4 | : | 2 6 7 8 |
| 12 | 5 | : | 1 2 3 6 9 |
| 13 | 0 | : | |

The complemented local graph for orbit 6 is:

| | | | |
|---|---|---|---|
| 1 | 3 | : | 4 6 7 |
| 2 | 4 | : | 4 5 8 9 |
| 3 | 5 | : | 5 6 7 8 12 |
| 4 | 5 | : | 1 2 7 8 12 |
| 5 | 4 | : | 2 3 7 10 |
| 6 | 3 | : | 1 3 8 |
| 7 | 5 | : | 1 3 4 5 13 |
| 8 | 5 | : | 2 3 4 6 13 |
| 9 | 2 | : | 2 10 |
| 10 | 2 | : | 5 9 |
| 11 | 0 | : | |
| 12 | 2 | : | 3 4 |
| 13 | 2 | : | 7 8 |

## 1.2 scalar product between elements of orbits

This subsection made sense only if the representation is orthogonal which is the usual case Considering Orbit 1

1. Orbit Size is 5.
2. Norm of elements is 6.
3. Possible scalar product : 3

Considering Orbit 2

1. Orbit Size is 10.
2. Norm of elements is 27.
3. Possible scalar product : 21, 24

Considering Orbit 3

1. Orbit Size is 15.
2. Norm of elements is 8.
3. Possible scalar product : 6, 7

Considering Orbit 4

1. Orbit Size is 12.
2. Norm of elements is 25.
3. Possible scalar product : 20, 22, 23

Considering Orbit 5

1. Orbit Size is 60.
2. Norm of elements is 85.
3. Possible scalar product : 67, 68, 70, 71, 73, 74, 76, 77, 79, 80, 83

Considering Orbit 6

1. Orbit Size is 30.
2. Norm of elements is 129.
3. Possible scalar product : 96, 102, 105, 108, 120

### 1.3 invariant group of Orbits

The invariant group of $x$ is the set of $g$ such that $g(x) = x$ denoted by $G_x$.
If $x$ and $y$ are in the same orbit then the groups $G_x$ and $G_y$ are conjuguate.
For any element $x$ we denote $O_x$ its Orbit, we have $|O_x| \times |G_x| = |G|$ We print only nontrivial groups
Invariant group of Orbit 1:

1. Group Size is 24.
2. Elements are : (4 3 2 1 5 ), (4 3 1 2 5 ), (4 2 3 1 5 ), (4 1 3 2 5 ), (4 2 1 3 5 ), (4 1 2 3 5 ), (3 4 2 1 5 ), (3 4 1 2 5 ), (2 4 3 1 5 ), (1 4 3 2 5 ), (2 4 1 3 5 ), (1 4 2 3 5 ), (3 2 4 1 5 ), (3 1 4 2 5 ), (2 3 4 1 5 ), (1 3 4 2 5 ), (2 1 4 3 5 ), (1 2 4 3 5 ), (3 2 1 4 5 ), (3 1 2 4 5 ), (2 3 1 4 5 ), (1 3 2 4 5 ), (2 1 3 4 5 ), (1 2 3 4 5 )

Invariant group of Orbit 2:

1. Group Size is 12.
2. Elements are : (3 2 1 5 4 ), (3 1 2 5 4 ), (2 3 1 5 4 ), (1 3 2 5 4 ), (2 1 3 5 4 ), (1 2 3 5 4 ), (3 2 1 4 5 ), (3 1 2 4 5 ), (2 3 1 4 5 ), (1 3 2 4 5 ), (2 1 3 4 5 ), (1 2 3 4 5 )

Invariant group of Orbit 3:

1. Group Size is 8.
2. Elements are : (1 5 4 3 2 ), (1 5 4 2 3 ), (1 4 5 3 2 ), (1 4 5 2 3 ), (1 3 2 5 4 ), (1 2 3 5 4 ), (1 3 2 4 5 ), (1 2 3 4 5 )

Invariant group of Orbit 4:

1. Group Size is 10.
2. Elements are : (5 4 3 2 1 ), (5 3 4 1 2 ), (4 5 2 3 1 ), (3 5 1 4 2 ), (4 2 5 1 3 ), (3 1 5 2 4 ), (2 4 1 5 3 ), (1 3 2 5 4 ), (2 1 4 3 5 ), (1 2 3 4 5 )

Invariant group of Orbit 5:

1. Group Size is 2.
2. Elements are : (4 5 3 1 2 ), (1 2 3 4 5 )

Invariant group of Orbit 6:

1. Group Size is 4.
2. Elements are : (2 1 3 5 4 ), (1 2 3 5 4 ), (2 1 3 4 5 ), (1 2 3 4 5 )

# 2 The intrisic informations concerning the graph of facet rays of 22SMET5.ine

The graph considered is made of 20 elements.
The group acting on 22SMET5.ine is Group2hmet5
Under the group action we have in fact only 1 orbits to consider.
The diameter of the graph is 1.
The list of orbits is with adjacency and Size info:

F1: -2 0 1 0 1 0 0 1 0 0 | 19 | 20

The representation matrix is:

| | F1 | Adj. | Size |
|---|---|---|---|
| F1 | 19 | 19 | 20 |
| Size | 20 | | 20 |

## 2.1 The local graphs

The complemented local graph are:
We consider graph with cardinal inferior or equal to 20
The complemented local graph for orbit 1 is:

| | | |
|---|---|---|
| 1 | 0 | : |
| 2 | 0 | : |
| 3 | 0 | : |
| 4 | 0 | : |
| 5 | 0 | : |
| 6 | 0 | : |
| 7 | 0 | : |
| 8 | 0 | : |
| 9 | 0 | : |
| 10 | 0 | : |
| 11 | 0 | : |
| 12 | 0 | : |
| 13 | 0 | : |
| 14 | 0 | : |
| 15 | 0 | : |
| 16 | 0 | : |
| 17 | 0 | : |
| 18 | 0 | : |
| 19 | 0 | : |

## 2.2 scalar product between elements of orbits

This subsection made sense only if the representation is orthogonal which is the usual case Considering Orbit 1

1. Orbit Size is 20.
2. Norm of elements is 7.
3. Possible scalar product : $-2$, $1$, $4$

## 2.3 invariant group of Orbits

The invariant group of $x$ is the set of $g$ such that $g(x) = x$ denoted by $G_x$.
If $x$ and $y$ are in the same orbit then the groups $G_x$ and $G_y$ are conjuguate.
For any element $x$ we denote $O_x$ its Orbit, we have $|O_x| \times |G_x| = |G|$ We print only nontrivial groups Invariant group of Orbit 1:

1. Group Size is 6.
2. Elements are : (3 2 1 4 5 ), (3 1 2 4 5 ), (2 3 1 4 5 ), (1 3 2 4 5 ), (2 1 3 4 5 ), (1 2 3 4 5 )

# 3 The two incidence matrix

Incidence between Orbit of Extreme rays and Orbits of Facets

| | $F1$ | $Inc$ |
|---|---|---|
| $E1$ | 16 | 16 |
| $E2$ | 12 | 12 |
| $E3$ | 12 | 12 |
| $E4$ | 10 | 10 |
| $E5$ | 10 | 10 |
| $E6$ | 10 | 10 |

Incidence between Orbits of Facets and Orbits of Extreme Rays

| | $E1$ | $E2$ | $E3$ | $E4$ | $E5$ | $E6$ | $Inc$ |
|---|---|---|---|---|---|---|---|
| $F1$ | 4 | 6 | 9 | 6 | 30 | 15 | 70 |